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# ON A COMBINATORIAL INTERPRETATION OF THE BISECTIONAL PENTAGONAL NUMBER THEOREM

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***Dedicated to Prof. A.K. Agarwal on his*** $70^{th}$ ***Birth Anniversary***

**Abstract:** In this paper, we invoke the bisectional pentagonal number theorem to prove that the number of overpartitions of the positive integer $n$ into odd parts is equal to twice the number of partitions of $n$ into parts not congruent to 0, 2, 12, 14, 16, 18, 20 or 30 mod 32. This result allows us to experimentally discover new infinite families of linear partition inequalities involving Euler's partition function $p(n)$. In this context, we conjecture that for $k > 0$, the theta series

$$(-q;-q)_\infty \sum_{n=k}^{\infty} \frac{q^{\binom{k}{2}+(k+1)n}}{(q;q)_n} \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$$

has non-negative coefficients.

**Keyword and Phrases:** Partitions, overpartitions, pentagonal number theorem.

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## 1. Introduction

The $18^{th}$ century mathematician Leonard Euler discovered a simple formula for the limiting case $n \to \infty$ of the $q$-shifted factorial

$$(a;q)_n = \begin{cases} 1, & \text{for } n = 0, \\ (1-a)(1-aq)\cdots(1-aq^{n-1}), & \text{for } n > 0 \end{cases}$$